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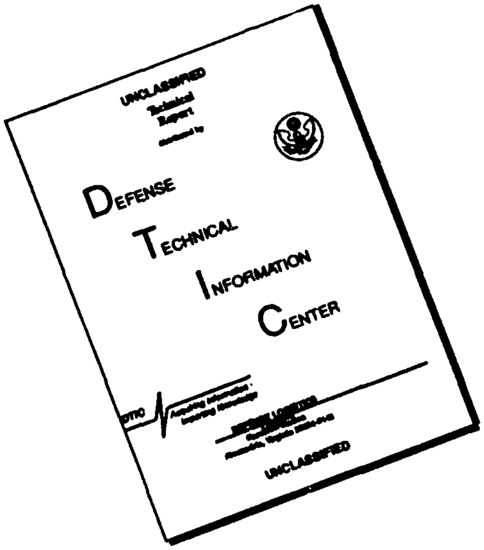

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RATIONAL APPROXIMATIONS WITH HANKEL-NORM CRITERION*

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Abstract

A two-variable approach to the model reduction problem with Hankel norm criterion is discussed. The problem is proved to be reducible to obtain a two-variable all-pass rational function, interpolating a set of parametric values at specified points inside the unit circle. A polynomial formulation and the properties of the optimal Hankel norm approximations are then shown to result directly from the general form of the solution of the interpolation problem considered. As a consequence, the recursive Nevanlinna algorithm can be employed and the essential stability properties of the solution can be established with the help of the Nevanlinna matrix [9]. This short paper is meant to briefly summarize the work in the full paper [8], where the reader is referred to for more details.

Introduction

In 1971, Adamjan, Arov and Krein [1] have obtained various powerful results related to the theory of bounded Hankel operators, which have great significance in the model reduction problem. Although their somewhat abstract original formulation in the framework of operator theory has been hindering their fast dissemination into the engineering community, the importance of this contribution has begun to be recognized. The relevance of [1] to the model reduction problem was first mentioned by Kung [2] in 1978, while some numerical aspects of the question were published in [3]. In [4],[5], connections between the minimal Hankel-norm approximations and the balanced state-space realizations introduced by Moore are put into light and shown to lead to an optimal approximation algorithm, requiring the solution of two Lyapunov equations and a singular value decomposition. In [6],[7], Kung took a pure one-variable polynomial approach and proved that the optimal approximation problem can be reduced to a simple generalized eigenvalue formulation when the transfer function is available.

A New Formalism for Rational Approximation

The problem of rational approximations in Hankel-norm can be defined as follows. Let a transfer function $h(z)$ or equivalently its impulse response be given and let $g(z)$ be some rational approximation of $h(z)$. Since the coefficients of the impulse response of a stable linear system are the Markov parameters of its transfer function, comparing two impulse responses amounts to comparing two Hankel matrices; the Hankel norm is precisely the spectral norm of the difference of these two matrices and it has a close relationship with other conventional norms [6]. The problem of the optimal approximations can then be stated in two different forms [6],[7],[8]:

a) for a preassigned tolerance, how to obtain a minimal order approximation of $h(z)$

b) for a given order, how to obtain the minimal norm approximation of $h(z)$

Clearly, the above two problems are intimately related.

It turns out that the problem of optimal approximations in Hankel norm can be globally approached and solved via a two-variable polynomial formalism [8].

A. Two-Variable Polynomial Formulation

Let - $h(z) = b(z)/a(z)$ be the original transfer function of order m.

- $\hat{a}(z) = z^n \overline{a}(1/\overline{z})$

- $r(\lambda,z)$, $\pi(\lambda,z)$ be two two-variable polynomials of degree n in z.

- $\hat{r}(\lambda,z) = z^n \overline{r}(\overline{\lambda},1/\overline{z})$

- ϵ can be any constant of unit modulus.

and let us consider the following equation:

$$\lambda b(z)r(\lambda,z) - \epsilon\, \hat{a}(z)\hat{r}(\lambda,z) = \pi(\lambda,z)a(z) \qquad (1)$$

The above equation clearly defines a linear system of equations in the unknown coefficients of the polynomials $r(\lambda,z)$ and $\pi(\lambda,z)$. It can be proved [8] that its minimal degree solution $r(\lambda,z)$, $\pi(\lambda,z)$ enjoys the following properties. For any fixed real value of λ and with $[\pi(\lambda,z)/\lambda r(\lambda,z)]_-$ standing for the stable projection of $\pi(\lambda,z)/\lambda r(\lambda,z)$ (i.e., after deleting its non-strictly stable part).

a) $[\pi(\lambda,z)/\lambda r(\lambda,z)]_-$ is a minimal order approximation of $h(z)$, whose Hankel norm does not exceed $|\lambda|^{-1}$.

b) If λ_{s+1}^{-1} is the $(s+1)^{th}$ singular value of the Hankel matrix associated with $h(z)$, and if one has the strict inequality $\lambda_s^{-1} > \lambda_{s+1}^{-1}$,

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$[\tilde{r}(\lambda_{s+1},z)/\lambda_{s+1} r(\lambda_{s+1},z)]_-$ is the minimal Hankel norm approximation of order s of h(z) and the value of the norm is precisely λ_{s+1}.

B. Parametric All-Pass Rational Interpolation

The above results can be easily proved with the help of standard matrix and polynomial techniques by a mere reformulation of the problem [8]. Let us indeed rewrite (1) in the following form:

$$\lambda \frac{b(z)}{a(z)} - \varepsilon \frac{\tilde{r}(\lambda,z)}{r(\lambda,z)} = \frac{\tau(\lambda,z)a(z)}{r(\lambda,z)\tilde{a}(z)} \qquad (2)$$

Let us assume the zeros of a(z) to be distinct $(z_1, z_2, \ldots, z_n)$ for the sake of brevity. With the notation $u_i = b(z_i)/\tilde{a}(z_i)$, solving (1) is clearly equivalent to obtain the minimal degree function $s(\lambda,z)=\varepsilon\tilde{r}(\lambda,z)/r(\lambda,z)$ interpolating the parametric values λu_i at the points z_i, i.e., $s(\lambda,z_i) = \lambda u_i$. As a matter of fact, the latter problem can be solved [1],[8] by the so-called Nevanlinna algorithm [9], which solves the interpolation problem in a recursive way. As a result, the two-variable polynomial $r(\lambda,z)$ can be constructed recursively and its stability properties investigated with the help of the Nevanlinna matrix, classically associated with the Nevanlinna algorithm [9]. Finally, the proof of the minimal order and minimal Hankel-norm properties of $[\tilde{r}(\lambda,z)/\lambda r(\lambda,z)]_-$ is completed from a direct link existing between the Nevanlinna matrix and the singular values of the Hankel matrix associated with h(z), [1], [8].

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